Poly-Many Hardcore Bits for Any One-Way Function and a Framework for Differing-Inputs Obfuscation

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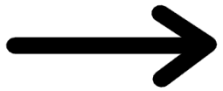
ASIACRYPT 2014

December 10, 2014

Classical Problem

Extract poly-many hardcore bits from any one-way function

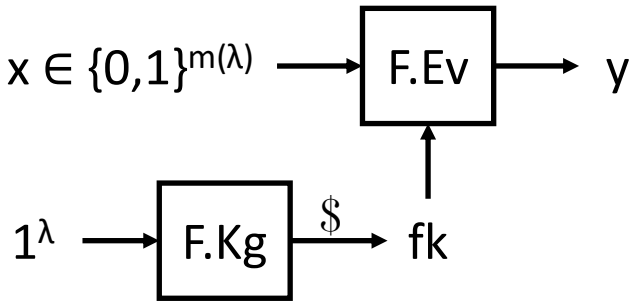


We describe the first solution

Main tool: indistinguishability obfuscation

Function Families

Function family F = (F.Kg, F.Ev)

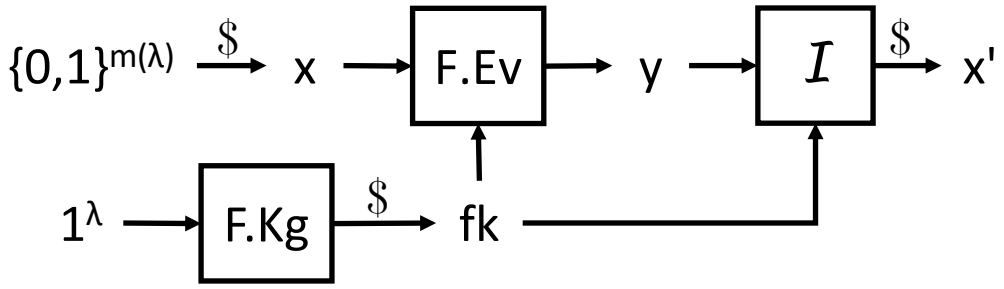


Example: RSA function family

fk = (N, e)

F.Ev(fk, x) = x^e mod N

One-Way Functions



$$\underline{\mathrm{OW}_{\mathsf{F}}^{\mathcal{I}}(\lambda)}$$
$$x \xleftarrow{\$} \{0,1\}^{m(\lambda)}$$
$$\mathrm{fk} \xleftarrow{\$} \mathsf{F.Kg}(1^{\lambda})$$
$$y \leftarrow \mathsf{F.Ev}(\mathrm{fk}, x)$$
$$x' \xleftarrow{\$} \mathcal{I}(\mathrm{fk}, y)$$
$$\text{Return } (\mathsf{F.Ev}(\mathrm{fk}, x') = y)$$

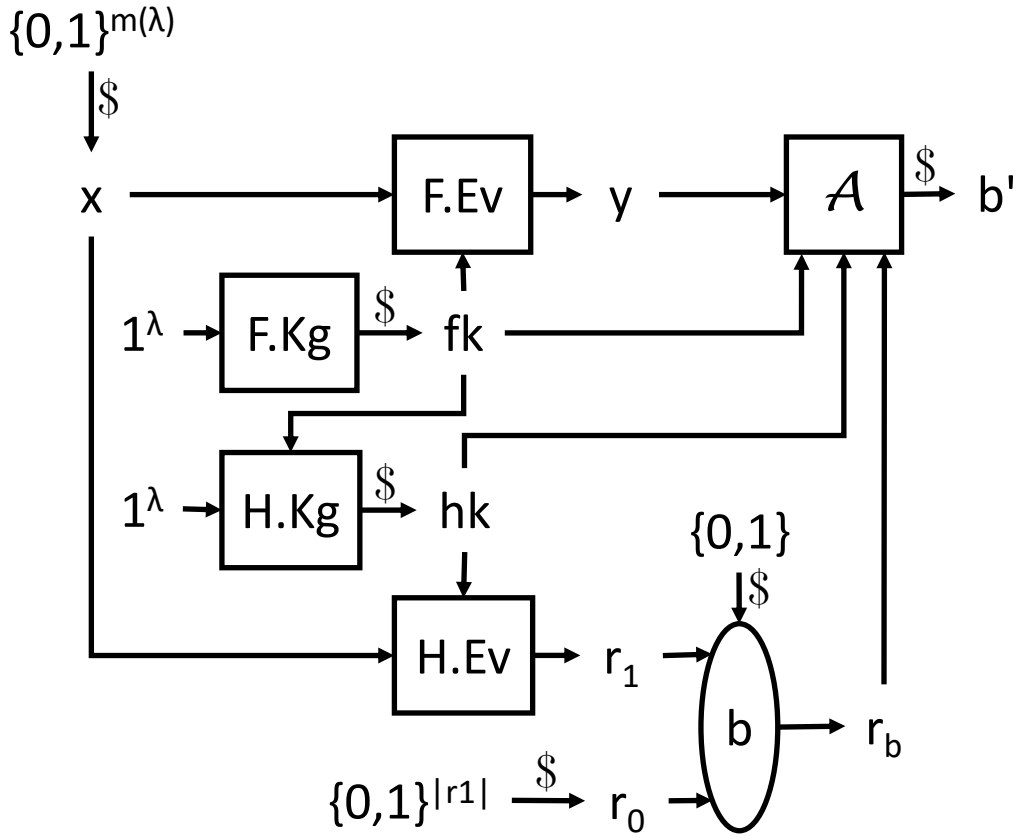
Function family F is one-way if

$$\forall \mathrm{PT}\ \mathcal{I}\colon \Pr\left[\mathsf{F.Ev}(\mathrm{fk}, x') = y\right]$$

is negligible

Hardcore Functions

Let F be a one-way function family



$\underline{\mathrm{HC}^{\mathcal{A}}_{\mathsf{F},\mathsf{H}}(\lambda)}$

$x \xleftarrow{\$} \{0,1\}^{m(\lambda)}$

$\mathrm{fk} \xleftarrow{\$} \mathsf{F.Kg}(1^\lambda);\ y \leftarrow \mathsf{F.Ev}(\mathrm{fk}, x)$

$\mathrm{hk} \xleftarrow{\$} \mathsf{H.Kg}(1^\lambda, \mathrm{fk});\ b \xleftarrow{\$} \{0,1\}$

If $b = 1$ then $r \leftarrow \mathsf{H.Ev}(\mathrm{hk}, x)$

else $r \xleftarrow{\$} \{0,1\}^{z(\lambda)}$

$b' \xleftarrow{\$} \mathcal{A}(\mathrm{fk}, \mathrm{hk}, y, r)$

Return $(b' = b)$

Function family H is hardcore for F if

$$\forall \mathrm{PT}\ \mathcal{A}\colon \Pr[b' = b] - 1/2$$

is negligible

Example: PKE scheme from RSA

RSA function family:

$fk = (N, e)$

$F.Ev(fk, x) = x^e \bmod N$

Conjecture: F is one-way

RSA public-key encryption scheme:

$pk = fk = (N, e)$

$Enc(1^\lambda, pk, m) = m^e \bmod N$

Not IND-CPA secure!

IND-CPA secure public-key encryption scheme from RSA:

$Enc_H(1^\lambda, pk, m) = (x^e \bmod N, H.Ev(hk, x) \oplus m)$, where:

indistinguishable from a uniformly random string

- H is hardcore for F
- $hk \xleftarrow{\$} H.Kg(1^\lambda, fk)$
- $x \xleftarrow{\$} \{0,1\}^{m(\lambda)}$

Limitation: how many bits can be encrypted?

$|H.Ev(hk, x)|$ – "span" of the hardcore function

We measure span as a function of $|x|$

History

One-Way Function	Hardcore Span	Required Assumption, Beyond One-Wayness	References
Any OWF (Goldreich-Levin hardcore bits)	1, log	–	[GL89]
RSA	poly	Φ-hiding; SSRSA	[LOS13]; [SPW06]
Rabin	poly	FSRSA	[SPW06]
Paillier	poly	BCCRA	[CGH01], [CGH02]
Discrete exponentiation modulo safe prime	poly	DLSE	[PS98], [MP05]
Discrete exponentiation modulo a composite	poly	Blum integer factorization	[HSS93], [GR03]
GPV lattice-based OWF [GPV08]	poly	LWE	[AGV09]
Niederreiter	poly	SDA + IA	[FGKRS10]
Any OWF	poly	UCE[$S^{cup} \cap S^{splt} \cap S^{one}$]	[BHK13]

Ad hoc solutions!

No polynomial span known for:

- discrete exponentiation modulo a prime
- discrete exponentiation in an elliptic curve group

Our Results

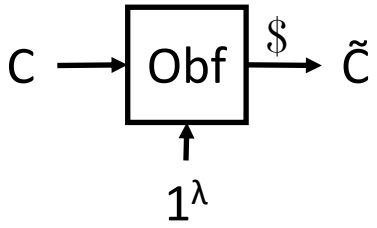
One-Way Function	Hardcore Span	Required Assumption, Beyond One-Wayness
Any injective OWF	poly	iO
Any OWF with polynomial pre-image size	poly	iO
Any OWF	poly	diO^{-}

Subsequent results:

One-Way Function	Hardcore Span	Required Assumption, Beyond One-Wayness	References
Any OWF	poly	iO + AIPO	[Brzuska-Mittelbach-14]
Any OWF	poly	Witness PRF	[Zhandry-14]

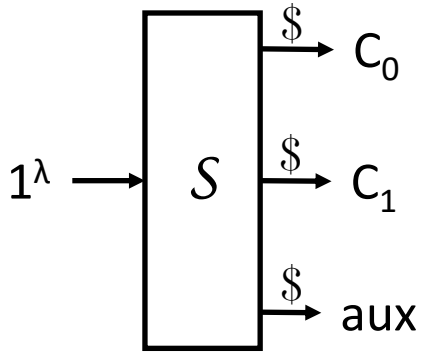
$\mathcal{S}$-secure Circuit Obfuscators

Circuit Obfuscators



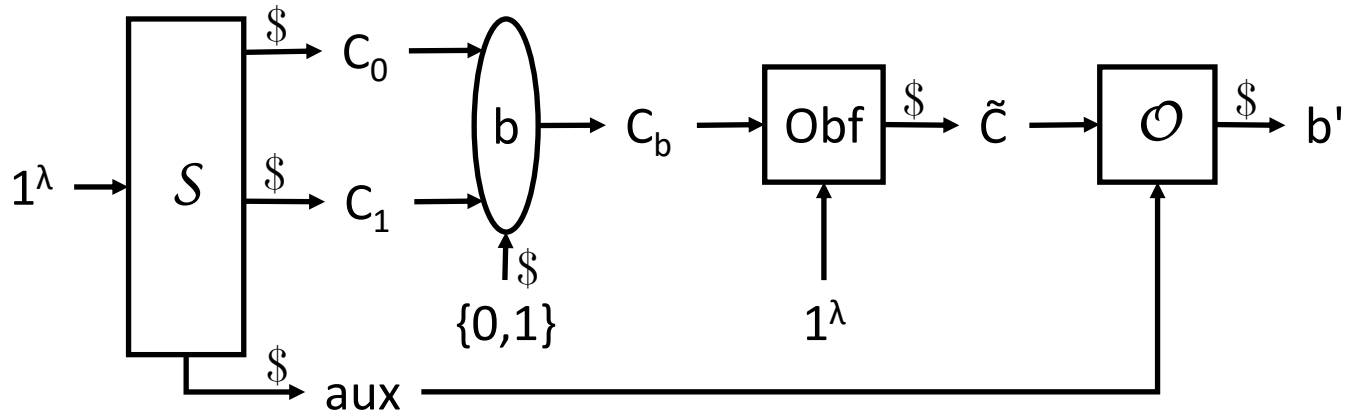
Correctness: $\forall x$: $C(x) = \tilde{C}(x)$

Circuit Samplers



Require: $|C_0| = |C_1|$

$\mathcal{S}$-secure Circuit Obfuscators



$\underline{\mathrm{IO}^{\mathcal{O}}_{\mathrm{Obf},\mathcal{S}}(\lambda)}$

$(\mathrm{C}_0, \mathrm{C}_1, aux) \xleftarrow{\$} \mathcal{S}(1^\lambda)$

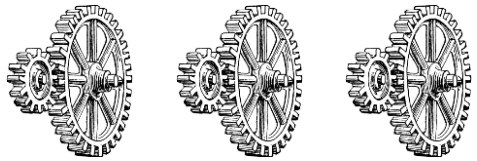
$b \xleftarrow{\$} \{0,1\}$

$\overline{\mathrm{C}} \xleftarrow{\$} \mathrm{Obf}(1^\lambda, \mathrm{C}_b)$

$b' \xleftarrow{\$} \mathcal{O}(\overline{\mathrm{C}}, aux)$

$\mathrm{Return}\ (b' = b)$

Let $\mathcal{S}$ be a class of circuit samplers

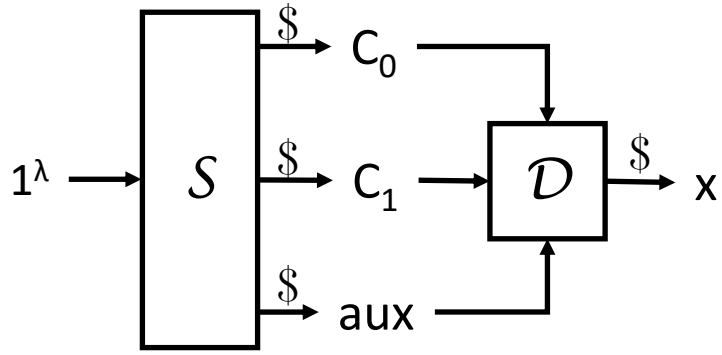


An obfuscator Obf is $\mathcal{S}$-secure if

$$\forall \mathrm{PT}\ \mathcal{O}\colon \Pr[b' = b] - 1/2$$

is negligible for all $\mathcal{S} \in \mathcal{S}$

Difference-Secure Circuit Samplers



$\underline{\mathrm{DIFF}_{\mathcal{S}}^{\mathcal{D}}(\lambda)}$

$(\mathrm{C}_0, \mathrm{C}_1, aux) \xleftarrow{\$} \mathcal{S}(1^\lambda)$

$x \xleftarrow{\$} \mathcal{D}(\mathrm{C}_0, \mathrm{C}_1, aux)$

Return $(\mathrm{C}_0(x) \neq \mathrm{C}_1(x))$

A circuit sampler $\mathcal{S}$ is difference-secure if

$$\forall \mathrm{PT}\ \mathcal{D}\colon \Pr\left[\mathrm{C}_0(x) \neq \mathrm{C}_1(x)\right]$$

is negligible

Let $\mathcal{S}_{\mathrm{diff}}$ be a class of difference-secure circuit samplers

Parametrized diO Framework

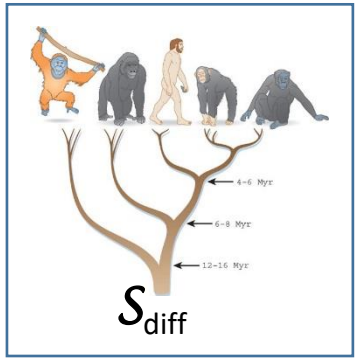
Abbreviation	Circuit Samplers	Definition
$\mathcal{S}_{\text{diff}}$	are difference-secure	see previous slide
$\mathcal{S}_{\text{eq}}$	produce equivalent circuits	C_0 and C_1 always agree on all inputs
$\mathcal{S}_{\text{diff}}(d)$	are difference-secure; produce d-differing circuits	C_0 and C_1 differ on at most $d(\lambda)$ inputs
$\mathcal{S}^{\text{sh}}$	have short auxiliary inputs	$\lvert aux \rvert < \lvert C_b \rvert$ for all $b \in \{0,1\}$

Definitions:

$\mathcal{S}_{\text{eq}}$-secure obfuscator is an indistinguishability obfuscator
$\mathcal{S}_{\text{diff}}$-secure obfuscator is a differing-inputs obfuscator

[Boyle-Chung-Pass-14]:

If d is a polynomial then any $\mathcal{S}_{\text{eq}}$-secure obfuscator
is also a $\mathcal{S}_{\text{diff}}(d)$-secure obfuscator.



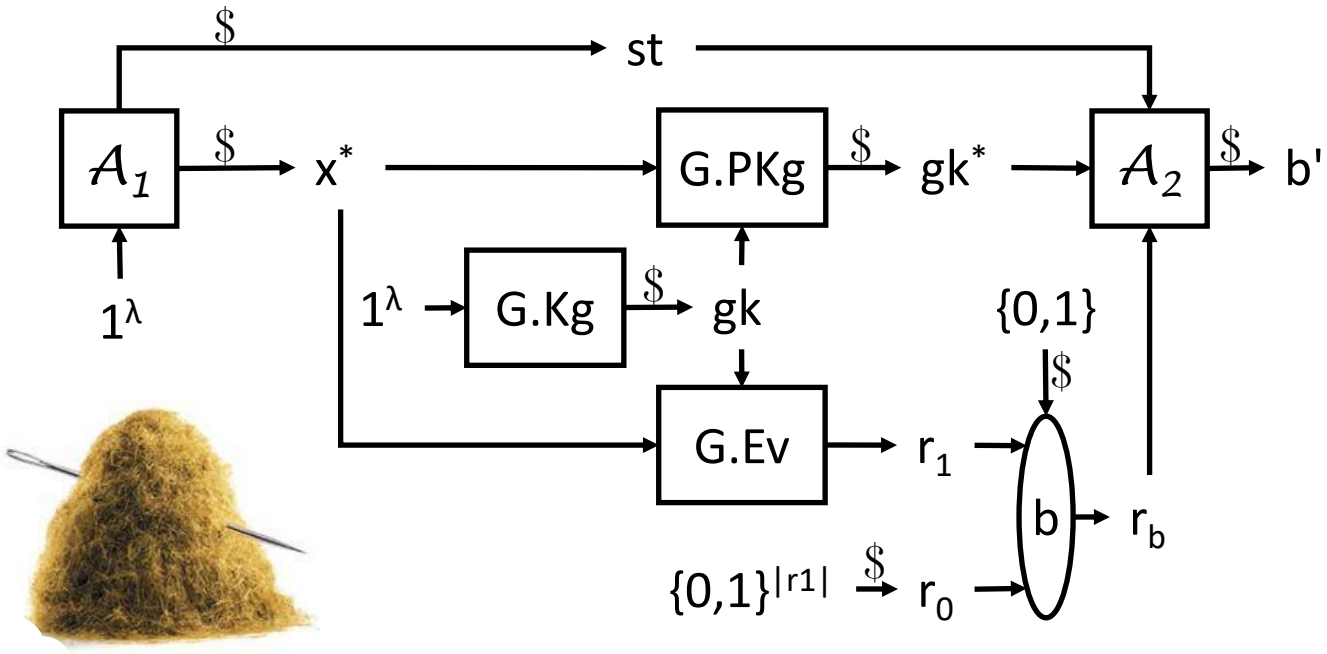
$\mathcal{S}_{\text{diff}}$

Punctured PRF

Let G be a function family extended with PKg

For any input x*: $gk^* \xleftarrow{\$} G.PKg(gk, x^*)$ such that

$$G.Ev(gk^*, x) = \begin{cases} G.Ev(gk, x), & x \neq x^* \\ \perp, & \text{otherwise} \end{cases}$$



$$\underline{\mathrm{PPRF}_{\mathsf{G}}^{\mathcal{A}}(\lambda)}$$
$$(x^*, \mathrm{st}) \xleftarrow{\$} \mathcal{A}_1(1^\lambda)$$
$$\mathrm{gk} \xleftarrow{\$} \mathsf{G.Kg}(1^\lambda)$$
$$\mathrm{gk}^* \xleftarrow{\$} \mathsf{G.PKg}(\mathrm{gk}, x^*)$$
$$b \xleftarrow{\$} \{0,1\}$$
$$\text{If } b = 1 \text{ then } r^* \leftarrow \mathsf{G.Ev}(\mathrm{gk}, x^*)$$
$$\text{else } r^* \xleftarrow{\$} \{0,1\}^{n(\lambda)}$$
$$b' \xleftarrow{\$} \mathcal{A}_2(\mathrm{st}, \mathrm{gk}^*, r^*)$$
$$\text{Return } (b' = b)$$

G is a punctured-PRF if

$$\forall \mathrm{PT}\ \mathcal{A}\colon \Pr[b' = b] - 1/2$$

is negligible

Our Result for Injective OWFs

Assume the following:

- Injective OWF F
- Punctured PRF G
- $S_{\text{diff}}(1)$-secure obfuscator Obf
  [BCP14]: S_{eq} is sufficient

Then there exists a polynomial s such that the function family

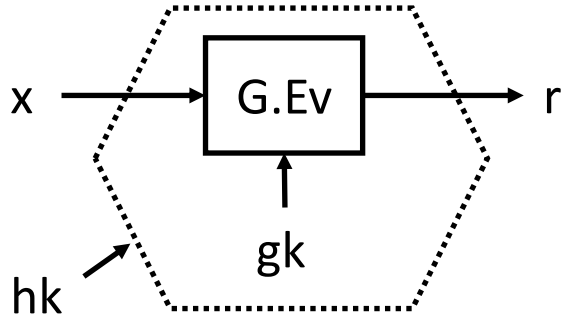
HC1[G, Obf, s] = (H.Kg, H.Ev)

is hardcore for F

$\underline{\mathsf{H.Kg}(1^\lambda, \mathrm{fk})}$
$\mathrm{gk} \xleftarrow{\$} \mathsf{G.Kg}(1^\lambda)$
$\mathrm{C} \leftarrow \mathsf{Pad}_{s(\lambda)}(\mathsf{G.Ev}(\mathrm{gk}, \cdot))$
$\overline{\mathrm{C}} \xleftarrow{\$} \mathsf{Obf}(1^\lambda, \mathrm{C})$
$\mathrm{hk} \leftarrow \overline{\mathrm{C}}$; Return hk

$\underline{\mathsf{H.Ev}(\mathrm{hk}, x)}$
$\overline{\mathrm{C}} \leftarrow \mathrm{hk}$
$r \leftarrow \overline{\mathrm{C}}(x)$
Return r

Note: polynomial s depends on F



Challenge: show that iO hides gk

Our Result for Any OWF

Assume the following:

- OWF F
- Polynomial d = PreImg_F
- Punctured PRF G
- ($\mathcal{S}_{\text{diff}}(d) \cap \mathcal{S}^{\text{sh}}$)-secure obfuscator Obf
  [BCP14]: $\mathcal{S}_{\text{eq}}$ is sufficient for polynomial d

Then there exists a polynomial s such that the function family

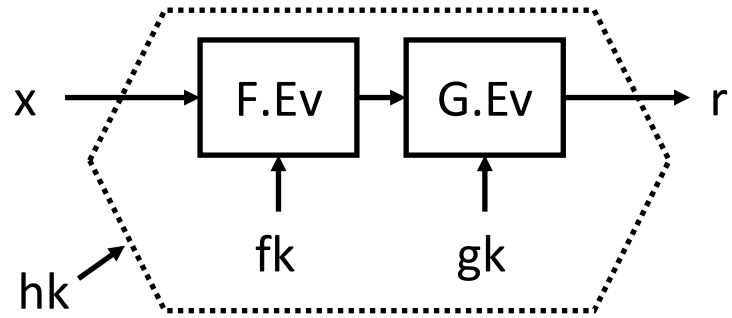
HC2[F, G, Obf, s] = (H.Kg, H.Ev)

is hardcore for F

$\underline{\mathsf{H.Kg}(1^\lambda, \mathrm{fk})}$
$\mathrm{gk} \xleftarrow{\$} \mathsf{G.Kg}(1^\lambda)$
$\mathrm{C} \leftarrow \mathsf{Pad}_{s(\lambda)}(\mathsf{G.Ev}(\mathrm{gk}, \mathsf{F.Ev}(\mathrm{fk}, \cdot)))$
$\overline{\mathrm{C}} \xleftarrow{\$} \mathsf{Obf}(1^\lambda, \mathrm{C})$
$\mathrm{hk} \leftarrow \overline{\mathrm{C}}$; Return hk

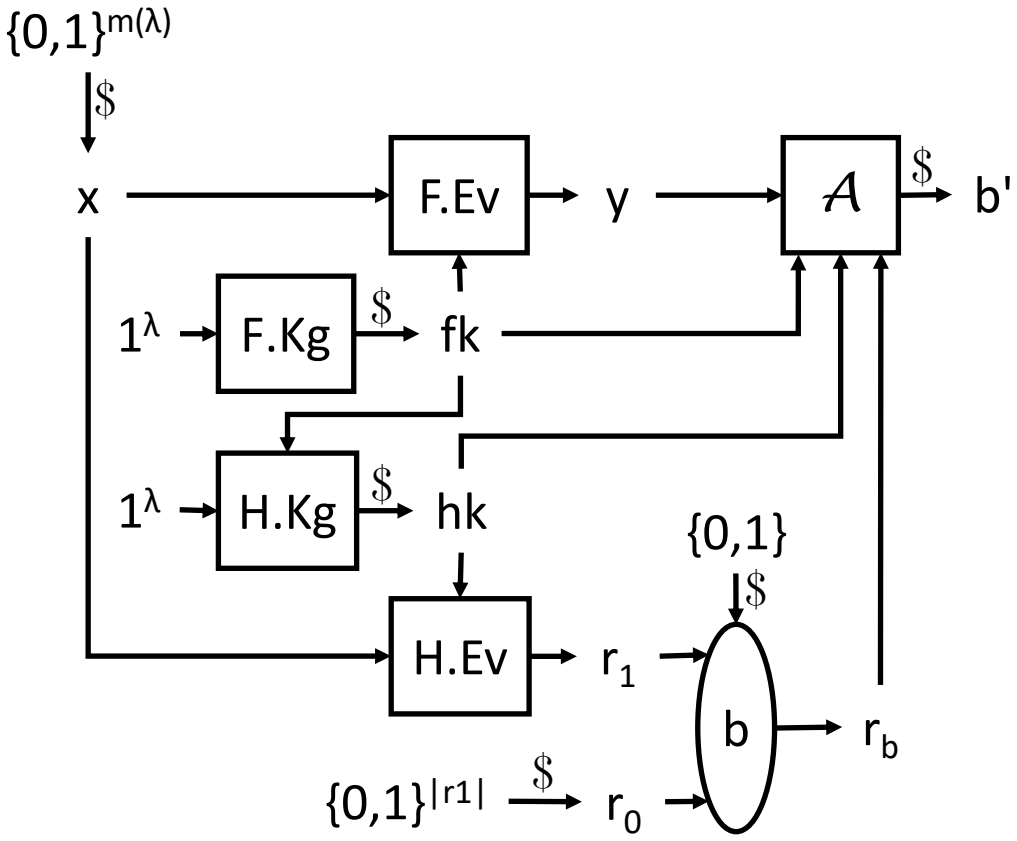
$\underline{\mathsf{H.Ev}(\mathrm{hk}, x)}$
$\overline{\mathrm{C}} \leftarrow \mathrm{hk}$
$r \leftarrow \overline{\mathrm{C}}(x)$
Return r

Note: H is "output-only dependent"



[GGHW14]: implausibility of diO and output-only dependent hardcore bits

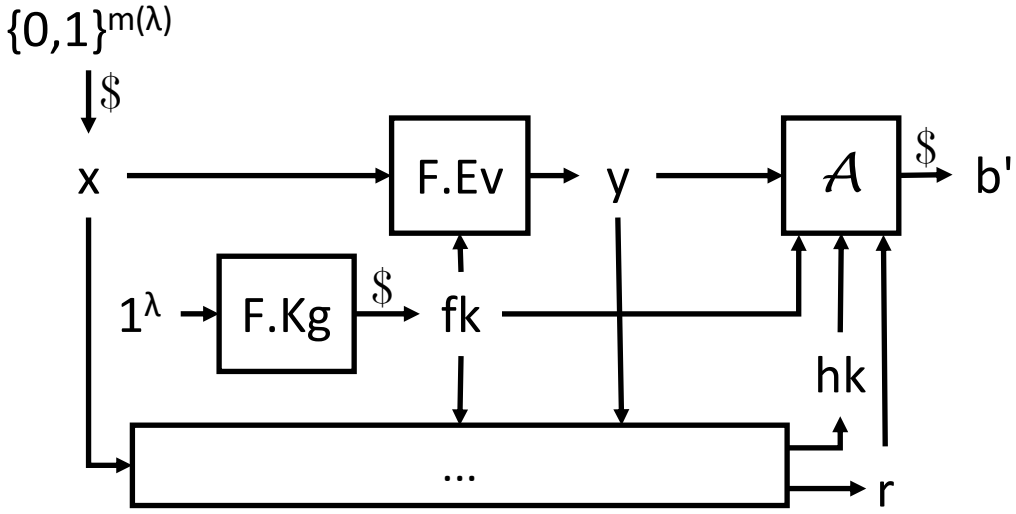
Injective OWFs: Proof Outline



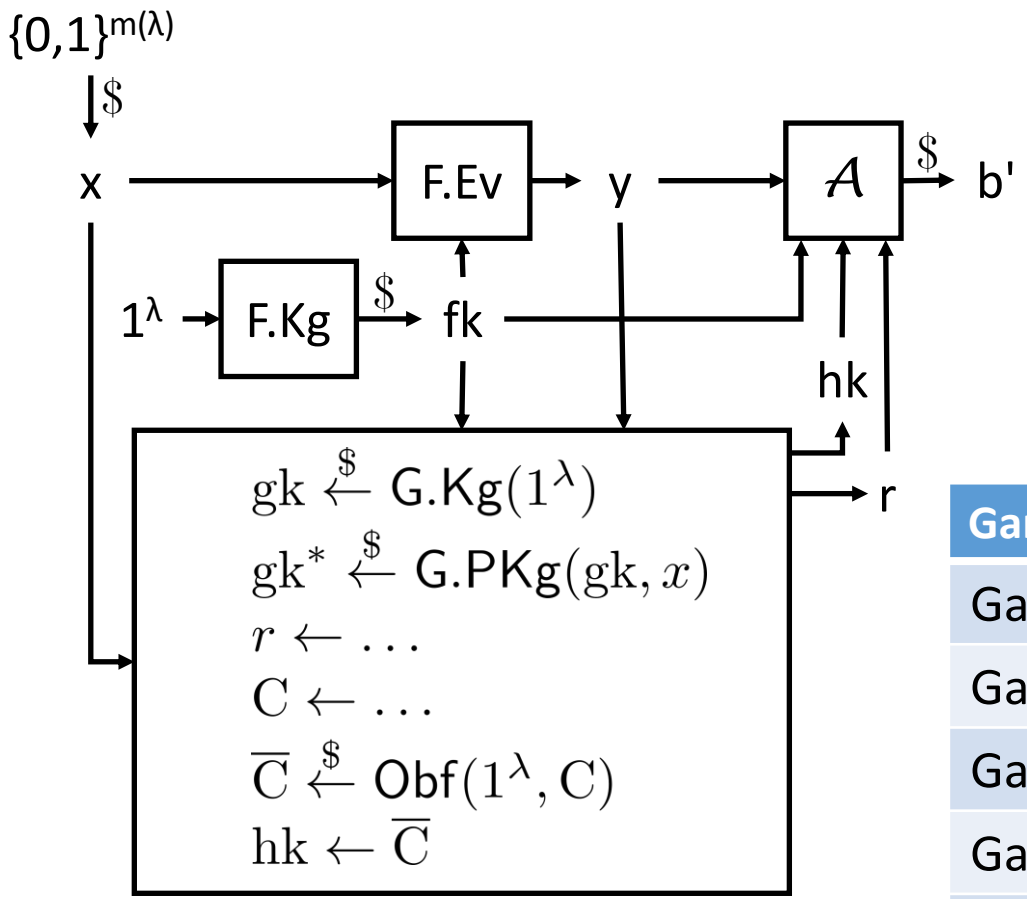
Hybrid games: $Game_0$, …, $Game_4$

$Game_0$ is equivalent to case b=1

$Game_4$ is equivalent to case b=0



Injective OWFs: Hybrid Games



Circuit $\mathrm{C}^{1}_{\mathrm{gk}^*,x,r}(x')$

If $x' \neq x$ then return $\mathsf{G.PEv}(\mathrm{gk}^*, x')$
Else return r

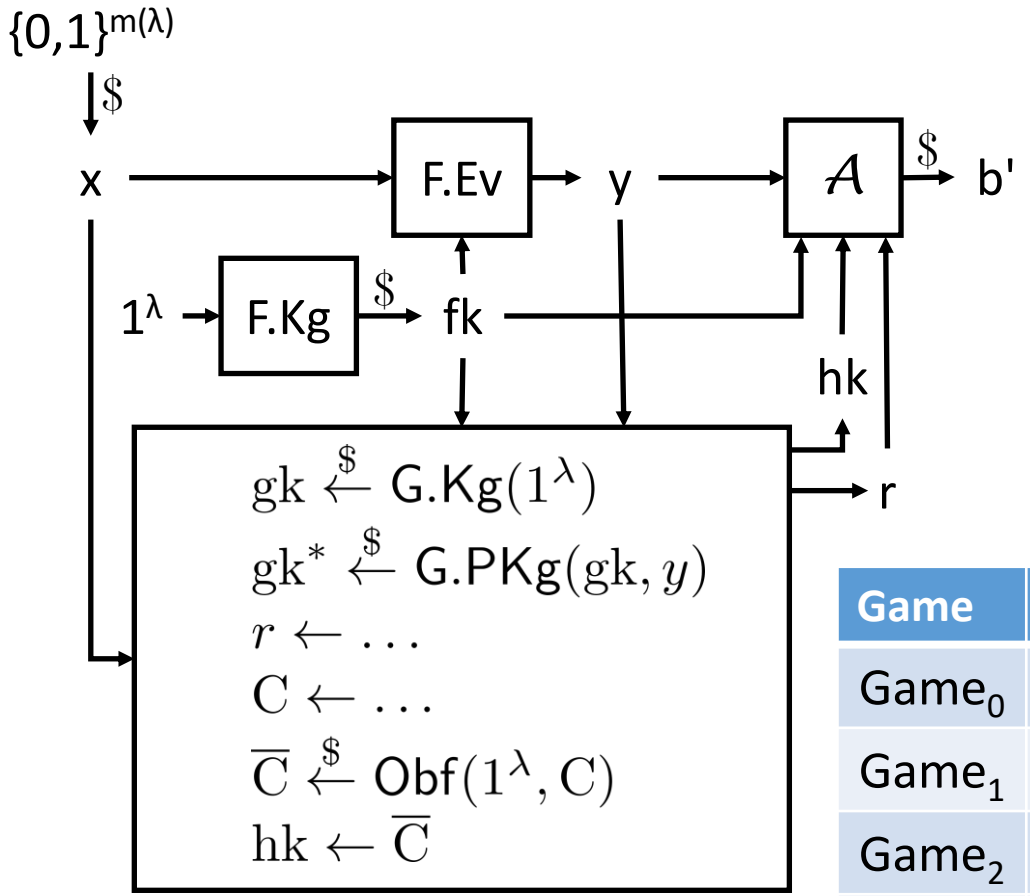
Circuit $\mathrm{C}^{2}_{\mathrm{fk},\mathrm{gk},y,r}(x')$

If $\mathsf{F.Ev}(\mathrm{fk}, x') \neq y$ then return $\mathsf{G.Ev}(\mathrm{gk}, x')$
Else return r

Game	r	C
Game_0	$r \leftarrow \mathsf{G.Ev}(\mathrm{gk}, x)$	$\mathrm{C} \leftarrow \mathsf{Pad}_{s(\lambda)}(\mathsf{G.Ev}(\mathrm{gk}, \cdot))$
Game_1	$r \leftarrow \mathsf{G.Ev}(\mathrm{gk}, x)$	$\mathrm{C} \leftarrow \mathsf{Pad}_{s(\lambda)}(\mathrm{C}^{1}_{\mathrm{gk}^*,x,r})$
Game_2	$r \xleftarrow{\$} \{0,1\}^{n(\lambda)}$	$\mathrm{C} \leftarrow \mathsf{Pad}_{s(\lambda)}(\mathrm{C}^{1}_{\mathrm{gk}^*,x,r})$
Game_3	$r \xleftarrow{\$} \{0,1\}^{n(\lambda)}$	$\mathrm{C} \leftarrow \mathsf{Pad}_{s(\lambda)}(\mathrm{C}^{2}_{\mathrm{fk},\mathrm{gk}^*,y,r})$
Game_4	$r \xleftarrow{\$} \{0,1\}^{n(\lambda)}$	$\mathrm{C} \leftarrow \mathsf{Pad}_{s(\lambda)}(\mathsf{G.Ev}(\mathrm{gk}, \cdot))$

iO
PPRF
iO + inj.
diO + OW

Any OWF: Hybrid Games



$$\frac{\text{Circuit } \mathrm{C}^{1}_{\mathrm{fk},\mathrm{gk}^*,y,r}(x')}{y' \leftarrow \mathsf{F.Ev}(\mathrm{fk}, x')}$$
$$\text{If } y' \neq y \text{ then return } \mathsf{G.PEv}(\mathrm{gk}^*, y')$$
$$\text{else return } r$$

Game	r	C	
Game_0	$r \leftarrow \mathsf{G.Ev}(\mathrm{gk}, y)$	$\mathrm{C} \leftarrow \mathrm{Pad}_{s(\lambda)}(\mathsf{G.Ev}(\mathrm{gk}, \mathsf{F.Ev}(\mathrm{fk}, \cdot)))$	iO
Game_1	$r \leftarrow \mathsf{G.Ev}(\mathrm{gk}, y)$	$\mathrm{C} \leftarrow \mathrm{Pad}_{s(\lambda)}(\mathrm{C}^{1}_{\mathrm{fk},\mathrm{gk}^*,y,r})$	PPRF
Game_2	$r \xleftarrow{\$} \{0,1\}^{n(\lambda)}$	$\mathrm{C} \leftarrow \mathrm{Pad}_{s(\lambda)}(\mathrm{C}^{1}_{\mathrm{fk},\mathrm{gk}^*,y,r})$	diO + OW
Game_3	$r \xleftarrow{\$} \{0,1\}^{n(\lambda)}$	$\mathrm{C} \leftarrow \mathrm{Pad}_{s(\lambda)}(\mathsf{G.Ev}(\mathrm{gk}, \mathsf{F.Ev}(\mathrm{fk}, \cdot)))$	

Hardcore Functions for Correlated Inputs

Assume the following:

- Arbitrary distribution D on d inputs
- Injective function family F one-way on D
- Punctured PRF G
- $\mathcal{S}_{\text{diff}}(d)$-secure obfuscator Obf
  [BCP14]: $\mathcal{S}_{\text{eq}}$ is sufficient for polynomial d

Then there exists a polynomial s such that the function family

HC1[G, Obf, s]

is hardcore for F with respect to D

Limitation: the size of hk grows with d

EwHCore construction by [Fuller-O'Neill-Reyzin-12]:

Yields a PRIV-secure D-PKE scheme for any input distribution

Limitation: the size of pk grows with the number of elements one can securely encrypt

Fin